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# Unit Commitment for Systems With Significant Wind Penetration

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**Abstract**—The stochastic nature of wind alters the unit commitment and dispatch problem. By accounting for this uncertainty when scheduling the system, more robust schedules are produced, which should, on average, reduce expected costs. In this paper, the effects of stochastic wind and load on the unit commitment and dispatch of power systems with high levels of wind power are examined. By comparing the costs, planned operation and performance of the schedules produced, it is shown that stochastic optimization results in less costly, of the order of 0.25%, and better performing schedules than deterministic optimization. The impact of planning the system more frequently to account for updated wind and load forecasts is then examined. More frequent planning means more up to date forecasts are used, which reduces the need for reserve and increases performance of the schedules. It is shown that mid-merit and peaking units and the interconnection are the most affected parts of the system where uncertainty of wind is concerned.

**Index Terms**—Power generation dispatch, power system economics, stochastic systems, wind power generation.

## I. INTRODUCTION

IN recent years, there has been a dramatic increase in the amount of wind power installed around the world, with further plans to increase the installed wind capacity in many countries, e.g., Denmark, Germany, Spain, Ireland [1], Great Britain [2], and many U.S. states [3]. This increase in installed wind capacity leads to various challenges for the operation of the power system, from frequency control issues [4], to planning of the transmission system [5]. One of the major challenges associated with wind energy is the way it impacts unit commitment. With low amounts of wind, unit commitment can be treated as a deterministic problem, as in [6]. However, large amounts of installed wind power add a significant stochastic element to the planning of the system. This is due to the uncertainty associated with wind power forecasts [7]. As wind cannot be forecast to a high degree of accuracy, extra reserve needs to be carried, in addition to the reserve already carried to cater for unit outages and demand forecast error, as shown in [8] and [9].

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By explicitly taking into account the stochastic nature of wind in the unit commitment algorithm, more robust schedules will be produced. Stochastic optimization has been used for unit commitment problems before, as in [10] and [11]. In [10], a long term security-constrained stochastic unit commitment (SCUC) model is described, which models unit and transmission line outages, as well as load forecasting inaccuracies. In [11], a method was developed to solve unit commitment problems when demand is not known with certainty. This approach uses multiple scenarios for demand. Both of these approaches show the benefits of using stochastic methods to solve the unit commitment problem. However, wind power as a stochastic input is not examined. Stochastic security with wind generation is examined in [12], which formulates a market-clearing problem capable of accounting for wind power. However, the concept of "rolling" over one year, explained later, is not examined in [12], while the system examined is small compared to the real system examined here. The WILMAR project [13] developed a stochastic scheduling tool to examine the impact of the variability of wind in energy markets. The system is rescheduled as more precise wind and load forecasts are made available, giving a "rolling planning" type of operation. Because more robust schedules are provided to cater for stochastic wind and load, the total expected costs of operating the system are lower than if a deterministic approach was used.

This paper examines several aspects of unit commitment that need to be considered when there are large amounts of wind on the system. Firstly, the benefits of using stochastic, instead of deterministic optimization to account for the uncertainty of wind in unit commitment are examined. Schedules produced with deterministic optimization are compared with stochastic results. These are also compared with results where perfect forecasting of wind and load is assumed. Initial analysis for the benefit of stochastic optimization with large wind penetration was carried out in [14] and [15]. The model used is updated for this paper and a more comprehensive and complete analysis is carried out. The second issue examined is the impact of modeling the uncertainty of wind in different timescales. More realistic amounts of uncertainty are included in the optimization by scheduling the system more frequently. The impact of modeling more of the uncertainty is examined. This shows the impact that more frequent rolling, using updated wind and load forecasts, has on the scheduling of power systems.

The methodology used is explained in detail in Section II. The test system used is outlined in Section III. The results are examined in Section IV, in terms of costs, the operation of units, interconnectors and performance of the schedules. Section V draws conclusions from the results.

## II. METHODOLOGY

### A. Model Used

The WILMAR model was originally used to study wind variability in the Nordic system, as described in [16]. This was then adapted to examine the Irish system as part of the All Island Grid Study [17]. What follows is a summary of the description of this updated model.

The main functionality of the WILMAR model is in two parts—the Scenario Tree Tool (STT) and the Scheduling Model. The STT is used to generate the scenarios that are used as inputs in the scheduling model. Possible future wind and load are represented by scenario trees, as shown in Fig. 1. The STT also produces time series for the forced unit outages. Each branch of the scenario tree corresponds to a different forecast of wind and load, as well as probability of occurrence. The required wind and load scenarios are generated by Monte Carlo simulations of the wind and load forecast error, based on an auto-regressive moving average model describing the wind speed forecast error. State of the art wind forecasting is assumed here. The high number of possible scenarios produced is then reduced using a scenario reduction approach, similar to [18]. Primary reserve, which is the reserve needed in shorter timescales, is estimated based on the largest in-feed to the system and the forecasted wind power production using results from [9]. Replacement reserve demand, which is the demand for reserve over longer timescales, is calculated based on the expected wind and load forecast error, with a different replacement reserve target for each scenario. More detailed information about the Scenario Tree Tool can be found in [17] and [19].

The scheduling model used here is a mixed integer, stochastic optimization model [20]. This is a more advanced model compared to that described in [13] and [16], which did not use mixed integer programming. However, the concepts that were used in that work remain the same. A mathematical formulation of the problem is given in the Appendix. It should be noted that this is the same as in [17], and contains much of the same formulae as found in [16]. This work is concerned with using these existing models to examine methods of dealing with uncertainty and the impact of uncertainty on unit commitment. The objective function being minimized, given in (A1), is the expected cost of the system over the optimization period, covering all of the scenarios, Fig. 1. This covers fuel costs, carbon costs and startup costs. This is subject to constraints on units, such as startup time, minimum up and down times (A7)–(A9), ramping rates (A6), and minimum and maximum generation [(A10), as well as interconnection constraints and losses, spinning and replacement reserve targets (A4) and (A5)], and penalties for not being able to meet load or reserve targets. The scheduling model has foresight of the scheduled outages of units, but not the forced outages produced in the STT. The objective function, the balancing equations and constraints and further explanation can be found in the Appendix. The Generic Algebraic Modeling System (GAMS) was used to solve the unit commitment problem using the mixed integer programming feature of the optimization software Cplex. More details about solve times and precision used are given in Section IV.

Rolling planning is shown in Fig. 1, in the case of rolling every 3 h. Starting at noon, the system is scheduled over 36

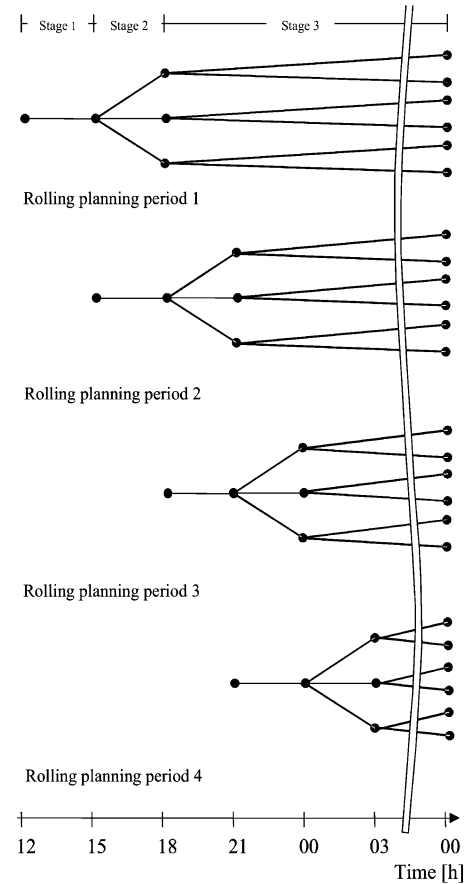


Fig. 1. Rolling planning with scenario trees.

h until the end of the next day. Subsequent planning periods take into account this day-ahead schedule, which is described in (A2). Schedules are updated to take into account changes in wind, load and available units from one planning period to the next. This happens in the intra-day balancing as described in (A3), whereby units are up and down regulated in relation to the day-ahead schedule. The commitment of the units, on or off, can also be changed intra-day. When rolling forward, the state of the units at the end of the first stage of the previous optimization period are used as the starting state of the next optimization period, i.e., if rolling is done every 3 h, the state of a unit (on or off and how long it has been on or off for) at the end of hour three is used as the starting state for the next optimization. After rolling forward, the system is then planned until midnight of the following day, so that the system is optimized eight times over a 24-h period. The planning period therefore gets shorter in each planning loop until noon of the following day when the period becomes 36 h again. The forecasts in the first stage, which is 3-h long in Fig. 1, are assumed to be perfect, representing “here-and-now” decisions, as can be seen by the fact that only one scenario is forecasted. This is due to the fact that a decision needs to be made about the exact operation of units in the first stage, as it represents realized values of wind and load—i.e., the actual operation of the system. The other two stages can be optimized using a “wait-and-see” approach, where there is a chance to change the schedule for this period in later optimizations.

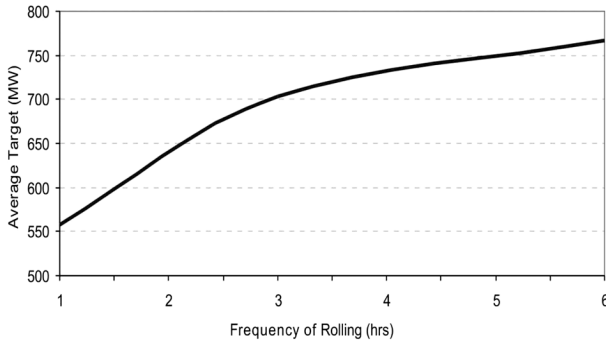


Fig. 2. Average replacement reserve versus frequency of commitment based on 90th percentile of wind and load forecast error.

### B. Cases Examined

Three different modes of optimization were examined—perfect, stochastic and deterministic. For each of these modes, solutions were found for three different rolling frequencies, meaning nine different cases were examined using the test system, which is described in the next section.

1) *Effect of Frequency of Rolling*: As a perfect forecast is assumed in the first stage of the scenario tree for all cases, the costs in this stage of the optimization are, on average, underestimated compared to the real costs that would be observed. Only the cost of uncertainty in later stages is modeled in the deterministic and stochastic modes described below. By shortening this first stage, more of the total uncertainty of wind will be included in the planning of the schedules, which will increase the cost of the planned schedules to more realistic levels. However, as this means more of the costs due to the uncertainty are minimized in the unit commitment, this would reduce costs when actually operating the system—this cannot be shown here, as only planned schedules are modeled. This has important implications for interpretation of results which will be highlighted later in Section IV-D. Planning the system more frequently has the effect of shortening this first stage, as the length of the “here-and-now” decisions shorten. It also has the effect of reducing the demand for replacement reserve on the system. Fig. 2 shows the change in replacement reserve versus frequency of commitment. As the first stage is shortened, the average demand for replacement reserve would decrease, as more frequent updating means more accurate forecasts are used and less replacement reserve is needed. To examine the effect that frequency of commitment and inclusion of more realistic uncertainty has, three different frequencies of commitment were examined for each mode (1, 3, or 6 h).

2) *Modes of Optimization*: Three different modes of optimization were examined. Each used mixed integer optimization.

- The *perfect* mode is used as the base case against which the other two modes are compared. Here, it is assumed that the wind and load can be perfectly forecasted. Therefore, each stage contains only one scenario, and this is the one that will be realized. Rolling planning is still carried out, so that the results are consistent with the other modes. Forced outages still occur, as they do in the other modes, and therefore rolling planning is needed to adjust the schedule in the next rolling planning period after forced outages occur. No extra reserve is carried to cater for wind and load forecast

errors. However, reserve is carried for the forced outage of the largest online unit.

- The *stochastic* mode uses the full scenario tree as explained earlier. Spinning reserve margins are kept so that all forecasted scenarios of wind and load are covered. By rolling more frequently, more of the uncertainty of wind and load is modeled. The first stage is still assumed to have perfect foresight, but multiple scenarios are modeled for later stages. Replacement reserve is carried to cover each scenario. The optimization is carried out over multiple possible scenarios, taking into account the probability of each occurring, so that the lowest expected cost solution is found.
- The *deterministic* mode has one scenario in each stage, as in the perfect mode. As with the stochastic and perfect modes described earlier, it assumes perfect foresight in the first stage. However, for the second and third stage, what is described as the “wait-and-see” stage earlier, there is only one scenario, as opposed to the multiple scenarios given in the scenario tree. This is found by taking the expected value of wind and load from the stochastic scenario tree. By multiplying the probability of a scenario occurring by the wind forecast in the scenario, and then adding all scenarios together, the expected value of wind is found. This will be different from the wind and load that will be realized, which is what makes the deterministic mode different from the perfect mode. To cater for this error, additional spinning and replacement reserve is carried, as described in the stochastic mode section. This deterministic solution is again carried out using rolling planning. The more frequently the system is planned, the more often the forecasts are updated, and therefore it would be expected that more accurate forecasts are used.

### III. TEST SYSTEM

To analyze the impact of large amounts of wind power on different aspects of unit commitment, a test system was examined. A possible plant mix for the Irish system in 2020 was chosen. The plant mix of this test system is based on one of the portfolios (portfolio 5) of the All Island Grid Study [21], derived using portfolio optimization method described in [22]. The All Island Grid Study was carried out to analyze the development of renewable energy on the Irish grid, and multiple possible portfolios were produced, with varying levels of installed wind power and conventional technologies. The particular portfolio has 6000 MW of installed wind power capacity, producing 18.4 TWh of wind energy over the year (which corresponds to approximately 34.3% of total energy demand—renewable energy makes up 42% of total energy demand in the portfolio chosen, due to tidal, hydro, and base renewables). The total installed conventional capacity on the system is approximately 8300 MW, including hydro units and base loaded renewables. This is made up of the units described in Table I, which groups multiple units according to fuel type. Note that two types of gas plant are included—mid-merit gas, i.e., open cycle gas turbines (OCGT) and aeroderivative gas turbines (ADGT), and base-loaded gas, i.e., combined cycle gas turbines (CCGT). Inflexible mid-merit plant here refers to the peat plant on the system—these use an indigenous fuel source classified as a type of brown coal [23].

TABLE I  
TYPES OF UNIT IN PLANT PORTFOLIO USED IN STUDY

Type of unit	No	Capacity (MW)	Fuel (€/GJ)
Base-loaded Gas	12	4114	5.91
Mid merit Gas, Peaking	19	1646	6.46
Coal	5	1257	1.75
Inflexible Mid Merit	3	345	3.71
Base RE	1	306	2.78
Hydro	1	216	-
Pumped Storage	4	292	-
Tidal	-	200	-
Wind Power	-	6000	-

TABLE II  
STARTUP TIME FOR CONVENTIONAL UNITS USED

Type of unit	Start-up time (hrs)
Mid merit Gas and Peaking	< 1
Inflexible Mid Merit	1-4
Base-loaded Gas	1-4
Coal	1-5

Peaking units, which use distillate, are shown here with mid-merit gas due to the fact that both are similarly flexible, when considered on an hourly time resolution. Table I also shows the fuel prices used for the various conventional plants, to give an indication of where that type of unit is on the merit order of the system. The price given in the table for the gas units is an average of the different prices used in the model for each month of the year, as given in [17].

Table II shows the startup time for the various types of conventional plant on the system. This can vary for each fuel type because of different characteristics for different plant with the same fuel. Here, flexible units are defined as those that can come online in less than 1 h. It can be seen that the inflexible mid-merit plant cannot start in less than 1 h, and therefore are classified as not flexible, as are base loaded gas and coal units. Data for wind, load and unit characteristics is taken from [24], and used with the STT to produce scenario trees for the scheduling model. The system modeled has a peak demand of 9600 MW and a minimum demand of 3500 MW in 2020. Interconnection to Great Britain is assumed to be 1000 MW. The Great Britain electricity system is modeled by grouping together similar units in blocks, so there are large blocks for nuclear, coal, CCGT, etc., with wind providing approximately 12% of electricity demand. Wind and load is assumed to be perfectly forecast in Great Britain. The interconnector is operated on a day-ahead basis only, i.e., import or export is fixed at noon every day for the following day, and cannot be altered intra-day, i.e., when the system rolls forward, the exchange scheduled on the interconnector can not be changed. The average replacement reserve for the system is shown in Fig. 2 for varying frequencies of rolling. This was calculated based on the percentile of total forecast error which most closely matches the current demand for replacement reserves on the Irish system, which was found to be 90%.

## IV. RESULTS AND DISCUSSION

The cases simulated are examined to identify and quantify the benefits of stochastic optimization and the effect of the frequency of commitment on systems with significant penetration of wind power. Firstly, the operation of the system—that is the production by units, the starting of units, and the operation of the interconnector is examined. The performance of the schedules, i.e., the ability to meet demand and reserve targets, is also analyzed. Finally, the impact that the change of system operation has on costs is examined.

The model was run for a year of demand and wind data, produced by the STT. Due to the stochastic methods applied, the solution time proved prohibitively long using high precision. The Cplex mixed integer solver was used with a computer with an Intel Core Duo 1.83-MHz processor with 1 GB of RAM. The model took approximately eight days to solve the stochastic case with hourly rolling for one year of data and a duality gap of 1%. This was the case which took the longest time to solve, as it had to solve 8772 stochastic optimizations (one for each hour in the year 2020, a leap year, except the first 12 h of January 1). The shortest case to solve, which was the case with the perfect foresight mode and rolling every 6 h, took approximately 3 h to solve on the same computer at the same duality gap. The results for one week at this precision were compared to a case where the duality gap used was 0.1%, which took significantly longer to solve than one week with a duality gap of 1%. It was found that the total costs obtained were within 0.02% of each other, with operation of the system very similar for both precisions, e.g., number of starts and production of units was similar. Therefore, it was decided to use the lower precision (1%) for the multiple yearly runs. This precision would mean that with realistic value of lost load (VOLL), there would be hours where load and reserve targets may not be met. Therefore, in the model, the VOLL was chosen to be extremely large—€ 300 000/MWh for demand not met, spinning reserve is valued at € 200 000/MWh, and replacement reserve at € 10 000/MWh. The stochastic model with 3-h rolling planning in the 36-h planning loop covering 36 h had 179 000 constraints and 167 000 variables of which 16 000 were integer variables. This is the same number as that used in the model in [17]. The addition of integer variables is the major difference between the model used here and in [17] and that used in [16]. The number of constraints here is the same as in [17].

### A. Impact on Unit Operation

The operation of the system changes depending on the way the uncertainty is treated. As much of the uncertainty of wind occurs hour to hour, most of the changes would be expected to occur with the flexible mid-merit gas and peaking units.

1) *Mode of Optimization:* The percentage change in production by unit type can be seen in Fig. 3 for stochastic and deterministic modes compared to the perfect forecasting mode. Mid-merit gas and peaking units are used more in both of the cases where wind is not forecast perfectly compared to the perfect case, as expected. This is due to the system having to respond to events different to those forecast. Optimizing deterministically results in increase in use of the more expensive mid-merit gas and peaking units compared to optimizing stochastically. This is expected due to the fact that deterministic optimization would produce less robust schedules, and have to call on these units

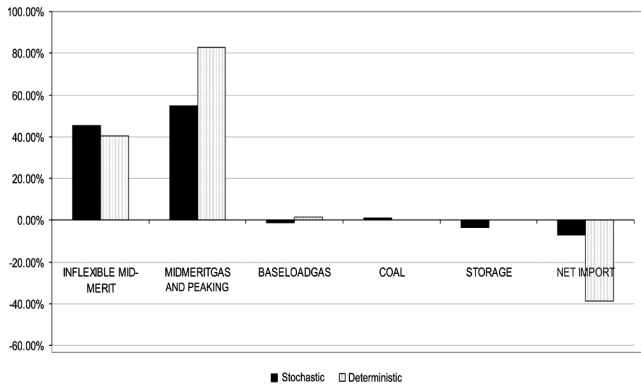


Fig. 3. Percentage change in production compared to perfect forecasting case for stochastic and deterministic cases (hourly rolling).

more. It should be noted that, when deterministically optimized every hour, mid-merit gas and peaking units still only provide approximately 1.5% of total production. The interconnector is used less in the cases where the wind is not perfectly known day-ahead, i.e., for the stochastic and deterministic cases. When the interconnector is planned day-ahead, the case with perfect foresight needs less replacement reserve than the cases with a forecast error. Therefore, when making the day-ahead plan, the cases with stochastic and perfect forecasting would plan differently—more units would be needed online. As these are already online to provide reserve, they will be used instead of the interconnector. The stochastic schedule makes more use of the interconnector than the deterministic schedule. Compared to the average wind power (and load) production scenario seen by the deterministic schedule, the low wind power production scenarios in the stochastic schedule increases production costs more than the scenarios with high wind power production due to the convexity of the supply curve. It is therefore optimal in the stochastic schedule to have higher imports than in the deterministic schedule due to the occurrence of low wind power production scenarios not seen by the deterministic schedule.

Fig. 4 shows the change in number of starts for the different modes of optimization compared to the perfect forecast case. An increased number of startups increases the startup costs—however, as it is total costs that are optimized, the optimal approach decided by the Cplex software in some hours would be to turn units on and off more frequently, thereby avoiding costs incurred when units are online and consuming fuel. It can be seen that including the forecast uncertainty causes all units to startup more frequently, as shown for both deterministic and stochastic cases when compared to the perfect case. It can also be seen that deterministic optimization results in increased starts compared to stochastic. This is due to the fact that less robust schedules mean more units will need to start to cater for forecast errors. The only units that are started more in the stochastic case are the inflexible mid-merit units, which are also producing more.

2) *Frequency of Rolling*: Fig. 5 shows the effect that changing the frequency of rolling has on the production of the units—the results shown are for the stochastic optimization. It can be seen, firstly, that the change in base-loaded units is small, showing that the impact of wind uncertainty on these units is minimal. Inflexible mid-merit units are being used more as the average replacement reserve targets increase, as these

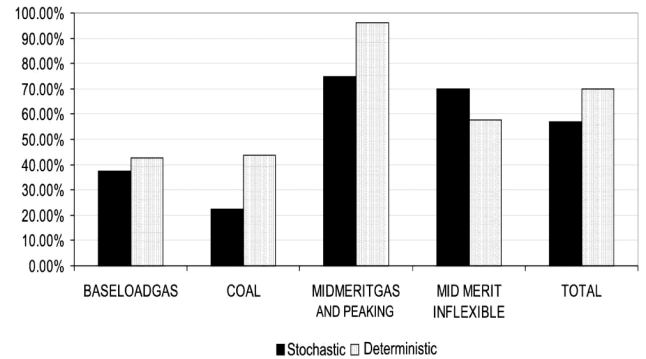


Fig. 4. Percentage change in startups compared to perfect forecasting case for stochastic and deterministic cases (hourly rolling).

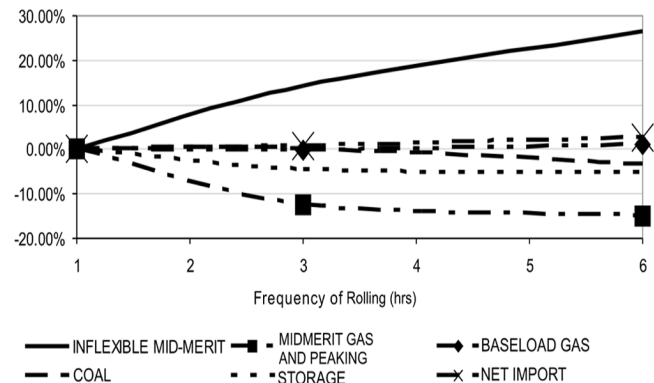


Fig. 5. Percentage change in production compared to hourly rolling case for stochastic optimization with varying frequencies of rolling.

cannot provide replacement reserve offline in less than 1 h, and therefore need to be online. Mid-merit gas, which can provide replacement reserve offline in less than 1 h, decreases its production as uncertainty on the system decreases (i.e., going from scheduling every 1 h to 6 h), as they are used more to deal with uncertainty due to their quick startup times and relatively low startup costs. Storage is used less as reserve increases, showing that it is being kept offline to provide this reserve.

Fig. 6 shows the number of startups obtained from the schedules for different frequencies of commitment. Firstly, it can be seen that the total number of startups decreases as less uncertainty is included in the model (i.e., going from 1 h to 6 h). As can be seen from the similar trend of the mid-merit gas and peaking units and the total system curve, mid-merit gas and peaking units make up the bulk of extra starts. These are the units that are generally started up most often on any system, due to their flexibility and position on the merit order. While the number of starts of inflexible mid-merit units is seen to increase going from 1 h to 6 h, these constitute a small percentage of the total number of startups (404 out of a total of 6558 in the hourly rolling case). However, Fig. 6, together with Fig. 5, show they are online more when reserve increases.

## B. Performance of Schedules

This section examines the impact on performance of the system, i.e., the ability of the schedules to meet demand, spinning and replacement reserve. As the way the uncertainty of wind is treated changes, i.e., whether deterministic or stochastic

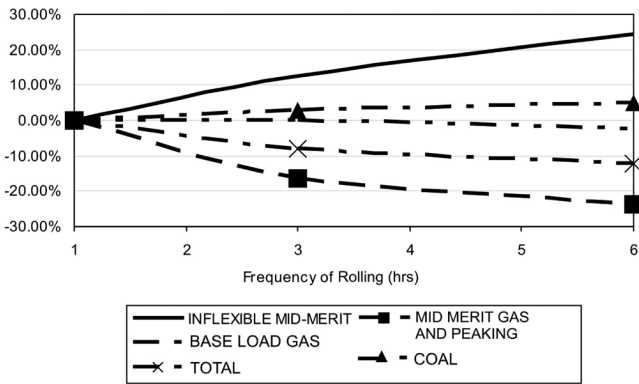


Fig. 6. Percentage change in startups compared to hourly rolling case for stochastic optimization with varying frequency of rolling.

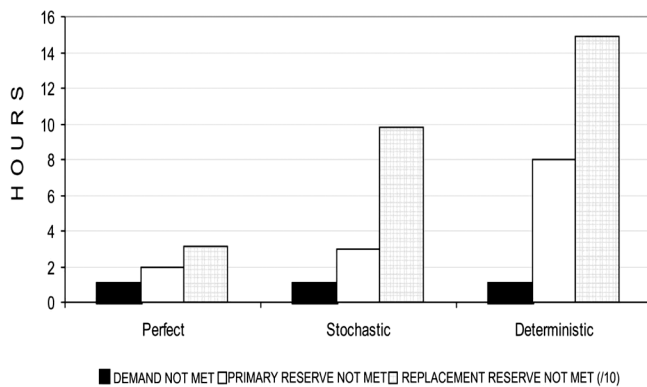


Fig. 7. Number of hours demand and reserve requirements not met over simulated year for different optimization modes, 1-h rolling. Note replacement reserve is divided by 10.

optimization is used, the ability of the system to meet load and reserve is affected. Better performing schedules will meet demand and reserve requirements more often.

When scheduling the system, there may be hours when the system cannot meet demand or reserve, due to lack of available capacity plus wind and interconnection in that hour. Fig. 7 compares the performance of the different modes of optimization in meeting demand and reserve. The number of hours demand cannot be met is seen to be equal regardless of mode of optimization, with demand for 1 h not being met in every case. This shows the performance of this particular plant mix over this particular year, and is different from measures such as loss of load expectation, which are based on probabilistic methods. This is for one realized wind and load time series, and one set of forced outages—if another time series was applied, a different performance might be observed. However, it can be seen that the perfect forecasting case performs best in meeting spinning and replacement reserve targets, followed by the stochastic solutions, with the deterministic solution performing worst, as expected.

Fig. 8 shows the number of hours demand and reserve requirements are not over the particular year simulated for varying frequencies of commitment using the stochastic mode. Again, it can be seen that the demand is not met once in every case. The number of hours reserve requirements are not met increases when moving from committing every hour to every 6 h. This would be expected, as the less often the system is committed, the less chance there is to account for the hours when there is

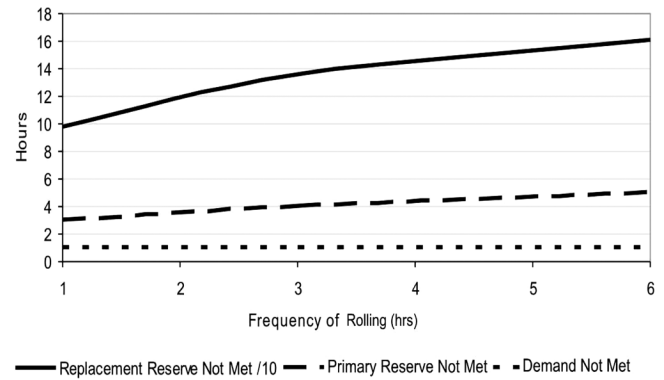


Fig. 8. Number of hours demand and reserve requirements not met over simulated year for different frequencies of rolling for stochastic mode. Note replacement reserve is divided by 10.

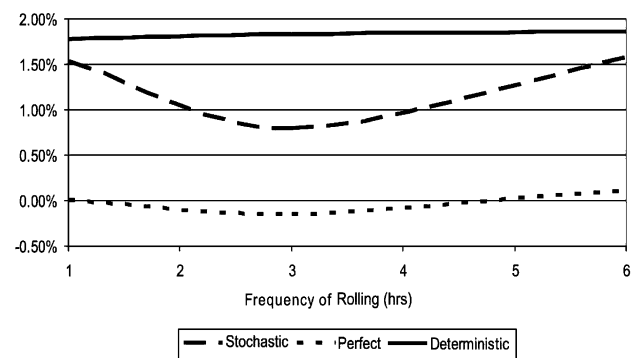


Fig. 9. Percentage change in costs compared to perfect forecasting case with hourly rolling.

the loss of a unit in the period from one planning period to the next.

### C. Impact on Costs

Fig. 9 shows the change in costs for the three modes examined for different rolling frequencies. These are the planned costs of both the island of Ireland and Great Britain. As wind and load in Great Britain is assumed to be perfectly forecasted, the only changes in the Great Britain system would be due to different wind and load forecasts in Ireland. Therefore, change in total costs is given as a percentage of Irish costs. The total costs given here are production costs, and do not include additional costs due to VOLL or value of lost reserve, which as stated earlier were made unrealistically high to ensure demand is met when possible.

Firstly, looking at the three different modes of optimization, it can be seen that the least costly mode is if perfect forecasting is assumed, as expected. This saves between 0.8% and 1.85% of costs for Ireland, depending on the mode being compared to and the frequency of rolling. However, as it assumes wind and load can be perfectly forecast, it is not a realistic result. By comparing the stochastic case with the deterministic case, it can be seen that a saving of approximately 0.25% (1-h rolling) to 0.9% (3 h rolling) can be made if the system is optimized stochastically as opposed to deterministically. It should be kept in mind that these two modes of optimization use the same forecasts, and only differ in how they deal with them—one mode optimizes over all forecasts, whereas the other optimizes for the

average expected value. This therefore shows the value of the stochastic approach. Note that this improvement in costs is different to the result obtained in [14] of 0.6%, and is due to the more accurate method of modeling provision of replacement reserve. Here, units can provide replacement reserve offline if they have a startup time less than 1 h, whereas in [14] it is assumed all replacement reserve is provided by online units, which is not as accurate a method of modeling replacement reserves.

The deterministic case does not change significantly in Fig. 9 as frequency of commitment changes—this is due to the fact that similar schedules will be produced as the deterministic optimization is carried out for one expected value of wind and load only. There is a slight increase in cost, due to increase in reserve demand as commitment frequency decreases. The costs for the perfect case can be seen to change slightly with varying frequencies of commitment. As wind and load is perfectly known in this case, it is only the change in the way unplanned unit outages are dealt with that causes this change in costs.

In the stochastic case, the changes can be seen to be different from what might be expected, with a minimum at approximately 3 h, and increasing as the frequency gets higher and lower from this point. This illustrates the fact that two different factors are accounting for the changes as the rolling planning frequency is changed. The first factor is due to the additional replacement reserve that is needed as the planning is carried out less frequently. Fig. 2 shows that, as the frequency of rolling decreases, the average demand for replacement reserve grows, as wind cannot be forecast as accurately at longer time horizons. This increase in reserve demand when rolling less often would be expected to cause an increase in system costs as more production capacity has to be reserved to provide replacement reserves. The other factor which influences the results is due to a modeling assumption. This is explained in more detail in the next section.

#### D. Modeling Assumption and Impact on Results

There is a modeling assumption that means care must be taken when interpreting the results shown in Fig. 9. This assumption, which is explained in Section II-A, is that the first stage of the scenario tree is assumed to have a perfect forecast. When the frequency of rolling changes, the length of this first stage changes—as it shortens, more of the uncertainty of wind can be accounted for. This increases the planned up-regulation and down-regulation of power plants when rolling more often as the length of the first stage, with perfect wind and load foresight, is reduced. This means mid-merit gas and peaking units are used more, as shown in Fig. 5.

To isolate the effect of this assumption on results, the model was changed so that all units could carry replacement reserve offline, no matter how long they take to startup. Therefore, the extra replacement reserve demand needed as the system is committed less often does not have an effect on the results. In reality this would not be true for this system, as it assumes short startup times for all units, but the effect of increasing the amount of the uncertainty included in the model can be isolated and its impact examined. The yearly simulation was re-run and the results obtained can be seen in Fig. 10. Here, the modeled costs can be seen to increase as the frequency of commitment increases towards hourly commitment in the stochastic case. This is as expected, as more of the uncertainty of wind is being modeled.

The results in Fig. 9 need to be interpreted in light of the characteristics illustrated in Figs. 2 and 10. It can be concluded from these that the changes in system costs in the model when changing the frequency of rolling planning are due to the two factors described previously. The first factor, the increase in demand for replacement reserve when rolling less often, dominates the change in costs when going from 3-h rolling to 6-h rolling. This is something that would be seen when operating a real power system. On the other hand, the increase in up- and down-regulation dominates the change in costs when going from 3-h rolling to 1-h rolling. This is a more realistic representation of the operation of a real power system, as more of the uncertainty is modeled. An additional cost would be seen in actual operation which is not modeled here, due to the fact that there would be uncertainty in the first stage. This unmodeled cost would be reduced as rolling is done more frequently in the model.

In conclusion, Fig. 9 illustrates that it is better to operate the system rolling every 3 h compared to every 6 h. This would be expected as more up to date information is being used in the optimization, and more replacement reserve would be needed when rolling less often, so this result would be seen in operation. However, increasing the rolling planning frequency shortens the perfect foresight stage, making the model more realistic as described above. Therefore, it cannot be concluded from Fig. 9 that it is better to roll every 3 h compared to every hour, as the change is due to a modeling issue. It would be expected that the opposite is true but the modeling limitations do not allow this conclusion to be drawn.

## V. CONCLUSION

This paper examined the impact of the stochastic nature of wind on planning and dispatch of a system. Examining the modes of optimization, it is shown that stochastic mode result in better performing and less costly schedules than deterministic optimization when the uncertainty of wind is taken into account. mid-merit and peaking plant are used less, and interconnection used more. More frequent scheduling of the system means wind and load forecasts are being updated more often and more of the uncertainty of wind is captured in the model. This means more of the costs due to uncertainty will be minimized, leading to more optimal results and better performing schedules.

## APPENDIX

### FORMULATION OF UNIT COMMITMENT PROBLEM

The formulation given below corresponds to the model presented in [17], which is based on work described in [16].

#### A. Nomenclature

##### 1) Indices:

DET	Deterministic region.
DISPATCH	Dispatchable units.
F	Fuel.
FAST	Units that can start in less than 1 h.
i,I	Unit group.



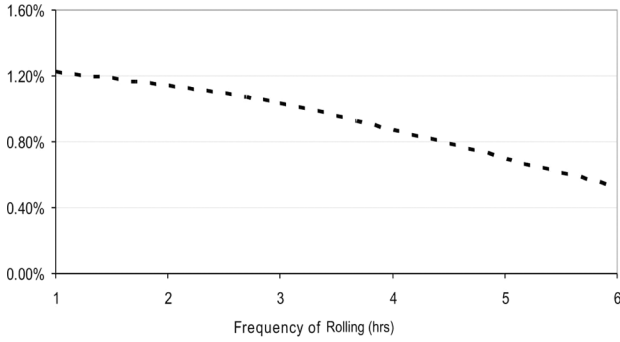


Fig. 10. Percentage change in costs versus frequency of rolling when only taking into account the effect of modeling more of the uncertainty (i.e., all units can provide reserve offline) for the stochastic case. Compared to base case of perfect forecasting with hourly rolling.

r, R	Region.
s, S	Scenario.
START	Units with startup fuel consumption.
STOCH	Stochastic region.
STOR	Units with pumped storage.
t, T	Time.
USEFUEL	Unit using fuel.

### 2) Parameters:

CAPACITY	Maximum capacity of unit.
BIDWIND	Day-ahead bid for wind.
d	Demand.
DETWIND	Deterministic wind.
EMISSION	Rate of emission.
END	Endtime of optimization period.
INITUP	Number of hours unit must be initially online due to its minimum uptime constraint.
k	Probability of scenario.
L	Infeasibility penalty.
LOAD	Penalty for loss of load.
NODE	Another node belonging to same stage.
OUTAGE	Loss of power due to forced outage.
PERIOD	Length of optimization period.
PRICE	Fuel price.
RAMPUP	Maximum ramp up rate.
REALIZED	Realized demand.
REP	Penalty for not meeting replacement reserve.
REPDEM	Replacement reserve demand.
SPIN	Penalty for not meeting spinning reserve.
SPINDEM	Spinning reserve demand.

STAR-TRAMP	Maximum startup ramping rate.
TAX	Emission tax.
UPTIME	Minimum up time of unit.
WIND	Realized wind.
XLOSS	Transmission loss.

### 3) Variables:

CONS	Fuel consumed.
DAYA-HEAD	Day-ahead power.
OBJ	Objective function.
ONLINE	Integer on/off for unit.
P	Power output.
Q	Unit pumping.
QDAY	Day-ahead demand not met.
QINTRA	Intra-day demand not met.
QREP	Replacement reserve not met.
QSPIN	Spinning reserve not met.
REPOFF	Replacement reserve provision offline unit.
REPON	Replacement reserve from online unit.
SPINRES	Spinning reserve provided by unit.
U	Relaxation variable.
V	Decision variable—on or off.
WINDCUR	Curtailed wind.
WINDRES	Wind curtailed for reserve.
+, -	Up, down regulation.

## B. Objective Function

The objective function being minimized is shown in (A1). The first part of (A1) is the operating fuel cost, and the second is the startup fuel cost (if a unit starts in that hour). The third line means that if a unit is online at the end of the day, the startup costs for it are subtracted from the objective function—this is to ensure that there are still units online at the end of the optimization period. The decision variable is given in the first three lines, showing whether a unit is online or offline. The fourth line is the costs due to emissions, while the last four lines describe the additional cost incurred due to penalties for not being able to meet load targets or reserve targets:

$$\begin{aligned}
 V_{\text{obj}} = & \sum_{i \in I^{\text{USEFUEL}}} \sum_{s \in S} \sum_{t \in T} k_s F_{i,r,s,t}^{\text{CONS}} F_{f,r,t}^{\text{PRICE}} V_{i,t}^{\text{ONLINE}} \\
 & + \sum_{i \in I^{\text{START}}} \sum_{s \in S} \sum_{t \in T} k_s F_{i,r,s,t}^{\text{START}} F_{f,r,t}^{\text{PRICE}} V_{i,t}^{\text{ONLINE}} \\
 & - \sum_{i \in I^{\text{START}}} \sum_{s \in S} k_s F_{i,r,s,T_{\text{END}}}^{\text{START}} F_{f,r,T_{\text{END}}}^{\text{PRICE}} V_{i,T_{\text{END}}}^{\text{ONLINE}}
 \end{aligned}$$

$$\begin{aligned}
& + \sum_{i \in I^{\text{USEFUEL}}} \sum_{s \in S} \sum_{t \in T} k_S F_{i,r,s,t}^{\text{CONS}} F_{f,r}^{\text{TAX}} F_f^{\text{EMISSION}} \\
& + \sum_{s \in S} \sum_{t \in T} k_S L^{\text{LOAD}} \left( U_{r,s,t}^{\text{QINTRA},+} + U_{r,s,t}^{\text{QINTRA},-} \right) \\
& + \sum_{t \in T} k_S L^{\text{LOAD}} \left( U_{r,t}^{\text{QDAY},+} + U_{r,t}^{\text{QDAY},-} \right) \\
& + \sum_{s \in S} \sum_{t \in T} k_S L^{\text{SPIN}} U_{r,s,t}^{\text{QSPIN},-} \\
& + \sum_{s \in S} \sum_{t \in T} k_S L^{\text{REP}} U_{r,s,t}^{\text{QREP},-}. \quad (\text{A1})
\end{aligned}$$

### C. Day-Ahead Balancing Equation

The day-ahead balancing equation (A2) is done at 12:00 on every day for the next 36 h. It uses deterministic values for wind and load to set day-ahead prices (based on marginal unit consuming fuel), plan the operation of the interconnector, and plan expected unit commitment:

$$\begin{aligned}
& \sum_{i \in I^{\text{DISPATCH}}} P_{i,r,t}^{\text{DAYAHEAD}} + i_{r,t}^{\text{TIDAL}} + i_{r,t}^{\text{RUNRIVER}} \\
& + i_{r_{\text{DET}},t}^{\text{DET WIND}} + i_{r_{\text{STOCH}},t}^{\text{BID WIND}} \\
& + (1 - \text{XLOSS}) P_{\bar{r},r,t}^{\text{TRANS}} \\
& = \left( U_{r,t}^{\text{QDAY},+} - U_{r,t}^{\text{QDAY},-} \right) + d_{r_{\text{DET}},t}^{\text{DET}} + d_{r_{\text{STOCH}},t}^{\text{BID}} \\
& + P_{r,\bar{r},t}^{\text{TRANS}}. \quad (\text{A2})
\end{aligned}$$

### D. Intra-Day Balancing Equation

The intra-day balancing equation (A3) is done every planning period, for all scenarios. The interconnector between regions is fixed, so is not used to balance the load and generation. There is also the ability to relax the constraint for balancing the intra-day equation—however, this incurs a penalty as shown in the objective function (A1). When pumped storage is generating, it is included as a dispatchable unit, while when pumping it is added to demand—this is included in the optimization so that it is pumping and generating at the optimal times:

$$\begin{aligned}
& \sum_{i \in I^{\text{DISPATCH}}} (P_{i,r,s,t}^+ - P_{i,r,s,t}^-) - P_{r_{\text{STOCH}},s,t}^{\text{WINDCUR}} - V_{r_{\text{STOCH}},t}^{\text{WINDRES}} \\
& = U_{r,s,t}^{\text{QINTRA},+} - U_{r,s,t}^{\text{QINTRA},-} + i_{r_{\text{STOCH}},t}^{\text{BID WIND}} \\
& + i_{r_{\text{STOCH}},t}^{\text{OUTAGE}} - i_{r_{\text{STOCH}},s,t}^{\text{WIND}} + d_{r_{\text{STOCH}},s,t}^{\text{REALIZED}} - d_{r_{\text{STOCH}},t}^{\text{BID}}. \quad (\text{A3})
\end{aligned}$$

### E. Spinning Reserve Inequality

The spinning reserve is based on the largest online unit plus a target based on the amount of wind forecast in each hour. When the pumped storage is pumping, it contributes to spinning reserve:

$$\begin{aligned}
& \sum_{i \in I^{\text{DISPATCH}}} V_{r_{\text{STOCH}},s,t}^{\text{SPINRES}} + \sum_{i \in I^{\text{STOR}}} Q_{i,r,s,t} + V_{r_{\text{STOCH}},t}^{\text{WINDRES}} \\
& \geq U_{r,s,t}^{\text{QSPINRES}} + i_{r,s,t}^{\text{SPINDEM}}. \quad (\text{A4})
\end{aligned}$$

### F. Replacement Reserve Inequality

Only units with startup times less than 1 h can provide replacement reserve offline. Online units can also provide this reserve if they have the spare capacity over and above the capacity used for generation and spinning reserve:

$$\begin{aligned}
& \sum_{i \in I^{\text{DISPATCH}}} V_{r,s,t}^{\text{REPON}} + \sum_{i \in I^{\text{FAST}}} V_{r,s,t}^{\text{REPOFF}} \\
& \geq U_{r,s,t}^{\text{QREPRES}} + i_{r,s,t}^{\text{REPDEM}} \\
& - \text{MAX} \left( 0, i_{r_{\text{STOCH}},t}^{\text{BID WIND}} - i_{r_{\text{STOCH}},s,t}^{\text{WIND}} - d_{r_{\text{STOCH}},t}^{\text{BID}} \right. \\
& \left. + d_{r_{\text{STOCH}},s,t}^{\text{REALIZED}} - P_{r_{\text{STOCH}},t}^{\text{OUTAGE}} \right). \quad (\text{A5})
\end{aligned}$$

### G. Constraints on Unit Operation in Model

There are constraints on the operation of units on the system. These include startup time, minimum up and down time, maximum and minimum power output and ramping rates being obeyed.

Equation (A6) ensures that ramping rates of a unit are obeyed. This states that the power output form a unit in one period cannot be greater than the power output in the previous period plus the maximum ramping rate of that unit, if the unit is online. A similar equation constrains the ramping down rate:

$$\begin{aligned}
& P_{i,r,t}^{\text{DAY AHEAD}} + P_{i,r,s,t}^+ - P_{i,r,s,t}^- \\
& - \left( P_{i,r,t-1}^{\text{DAY AHEAD}} + P_{i,r,s,t-1}^+ - P_{i,r,s,t-1}^- \right) \\
& + V_{r,s,t}^{\text{REPON}} \\
& \leq V_{i,r,s,t}^{\text{ONLINE}} V_{\text{RAMPUP}}. \quad (\text{A6})
\end{aligned}$$

Equations (A7)–(A9) give expressions for the minimum up-time of units. They are stochastic versions of equations given in [25]. Similar constraints are given for minimum down time. Equation (A7) is related to the initial status of the units—i.e., the initial number of periods the unit must be online. Equation (A8) is used for the subsequent periods to satisfy the minimum up time constraint during all the possible sets of consecutive periods. Equation (A9) ensures that if the unit starts up it stays online in the remaining timespan:

$$\sum_{t=1}^{\text{INITUP}} (1 - V_{i,r,s,t}^{\text{ONLINE}}) = 0, \forall i \in I \quad (\text{A7})$$

$$\begin{aligned}
& \sum_{n=t}^{t+\text{UPTIME}_i-1} V_{i,r,s}^{\text{ONLINE}}(n) \\
& \geq \text{UPTIME}_i (V_{i,r,t}^{\text{ONLINE}} - V_{i,r,t-1}^{\text{ONLINE}}) \\
& \forall I, \forall t = \text{INITUP}_i \\
& + 1 \cdots T_{\text{PERIOD}} - \text{UPTIME}_i + 1, \quad \forall s \in S \quad (\text{A8})
\end{aligned}$$

$$\begin{aligned}
& \sum_{n=t}^{T_{\text{PERIOD}}} [V_{i,r,s}^{\text{ONLINE}}(n) - (V_{i,r,s,t}^{\text{ONLINE}} - V_{i,r,s,t-1}^{\text{ONLINE}})] \geq 0 \\
& \forall I, \forall t = T_{\text{PERIOD}} - \text{UPTIME}_i + 2 \cdots T, \\
& \quad \forall s \in S. \quad (\text{A9})
\end{aligned}$$

Equation (A10) describes the constraint on maximum available capacity for each unit—a similar equation constrains the minimum available capacity

$$\begin{aligned}
& P_{i,r,t}^{\text{DAY AHEAD}} + P_{i,r,s,t}^+ - P_{i,r,s,t}^- \\
& + V^{\text{SPINRES}}_{i,r,s,t} + V^{\text{REPON}}_{i,r,s,t} \\
& \leq P_i^{\text{CAPACITY}} * V^{\text{ONLINE}}. \quad (\text{A10})
\end{aligned}$$

## REFERENCES

- [1] Delivering a Sustainable Energy Future for Ireland, Government White Paper, 2007. [Online]. Available: <http://www.dcmnr.gov.ie>.
- [2] Meeting the Energy Challenge, U.K. Government White Paper, 2007. [Online]. Available: <http://www.dti.gov.co.uk>.
- [3] 2007 Market Report, AWEA. [Online]. Available: <http://www.awea.org/>.
- [4] G. Lalor, A. Mullane, and M. O. Malley, "Frequency control and wind turbine technologies," *IEEE Trans. Power Syst.*, vol. 20, no. 4, pp. 1905–1913, Nov. 2005.
- [5] C. Bayfield, S. Wood, A. Hiorns, and C. Trikha, "Planning for the integration of on-shore renewable generation for the gb transmission system," in *Proc. CIGRE 2006*, Paris, France, 2008, paper C2-108.
- [6] R. Baldick, "The generalized unit commitment problem," *IEEE Trans. Power Syst.*, vol. 10, no. 1, pp. 465–475, Feb. 1995.
- [7] I. Marti *et al.*, Evaluation of Advanced Wind Power Forecasting Models—Results of the Anemos Project, 2006. [Online]. Available: <http://anemos.cma.fr>.
- [8] L. Soder, "Reserve margin planning in a wind-hydro-thermal power system," *IEEE Trans. Power Syst.*, vol. 8, no. 2, pp. 564–571, May 1993.
- [9] R. Doherty and M. O'Malley, "A new approach to quantify reserve demand in systems with significant installed wind capacity," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 587–595, May 2005.
- [10] L. Wu, M. Shahidepour, and T. Li, "Stochastic security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 800–811, May 2007.
- [11] S. Takriti, J. Birge, and E. Long, "A stochastic model for the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1497–1508, Aug. 1996.
- [12] F. Bouffard and F. Galiana, "Stochastic security for operations planning with significant wind power generation," *IEEE Trans. Power Syst.*, vol. 23, no. 2, pp. 306–316, May 2008.
- [13] Wind Power Integration in Liberalised Electricity Markets (Wilmar) Project. [Online]. Available: [www.wilmar.risoe.dk](http://www.wilmar.risoe.dk).
- [14] A. Tuohy, P. Meibom, and M. O. Malley, "Benefits of stochastic scheduling for power systems with significant installed wind power," in *Proc. 10th Int. Conf. Probabilistic Methods Applied to Power Systems (PMAPS)*, Mayagüez, PR, 2008.
- [15] B. Hasche, R. Barth, and D. Swider, "Effects of improved wind forecasts on operational costs in the German electricity system," in *Proc. Int. Conf. Energy and Environmental Modeling*, Moscow, Russia, Sep. 2007.
- [16] R. Barth, H. Brand, P. Meibom, and C. Weber, "A stochastic unit commitment model for the evaluation of the impacts of the integration of large amounts of wind power," in *Proc. 9th Int. Conf. Probabilistic Methods Applied to Power Systems*, Stockholm, Sweden, 2006.
- [17] P. Meibom, R. Barth, H. Brand, D. Swider, H. Ravn, and C. Weber, All Island Renewable Grid Study Workstream 2b—Wind Variability Management Studies, 2007. [Online]. Available: <http://www.dcmnr.gov.ie>.
- [18] J. Dupacova, N. Grwe-Kuska, and W. Rmisch, "Scenario reduction in stochastic programming: An approach using probability metrics," *Math. Program.*, vol. 95, no. 3, pp. 493–511, 2003.
- [19] R. Barth, L. Sder, C. Weber, H. Brand, and D. Swider, Deliverable D6.2 (b) Documentation Methodology of the Scenario Tree Tool, Institute of Energy Economics and the Rational Use of Energy (IER), Univ. Stuttgart. Stuttgart, Germany, 2006. [Online]. Available: [www.wilmar.risoe.dk](http://www.wilmar.risoe.dk).
- [20] P. Kall and S. W. Wallace, *Stochastic Programming*. Chichester, U.K.: Wiley, 1994.
- [21] High Level Assessment of Suitable Generation Portfolios for the All-Island System in 2020 All Island Renewable Grid Study—Workstream 2A, 2006. [Online]. Available: <http://www.dcmnr.gov.ie>.
- [22] R. Doherty, H. Outhred, and M. O'Malley, "Establishing the role that wind may have in future generation portfolios," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1415–1421, Aug. 2006.
- [23] International Energy Agency, IEA 2007 Coal Information.
- [24] All-Island Modelling Project, 2005. [Online]. Available: <http://www.allislandproject.org>.
- [25] M. Carrion and J. Arroyo, "A computationally efficient mixed-integer linear formulation for the thermal unit commitment problem," *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1371–1378, Aug. 2006.



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